

Asian Basket Options with Individually Capped Returns

A model is presented for pricing Asian basket options with individually capped returns. Crude Monte Carlo method is employed to value the derivatives. Each asset return is capped with a rate. Volatility smile is utilized in the calculation.

Let $S_i(t)$ be a price process of a given underlying asset i , and suppose we have a basket of n assets. Let $\{t_0 < t_1 < \dots < t_m\}$ be a set of reset dates and $T \geq t_m$ be a payoff settlement date. The price of each asset is recorded on the set of reset dates to obtain the price arithmetic average of A_i . The payoff of this European type call option at the settlement date is given by

$$N \times \max\left(0.0, \sum_{i=1}^n w_i \times \min(\text{cap}_i, (A_i - K_i)/K_i)\right)$$

where N is the notional principal, w_i is the weight of asset i in the basket, K_i is the strike for asset i , and cap_i is the cap on return for asset i .

Let t be the current value date, then the current value of this derivative security can be written as

$$df(t, T) \times N \times E_t \left[\max \left(0, \sum_{i=1}^n w_i \times \min(\text{cap}_i, (A_i - K_i) / K_i) \right) \right]$$

where $df(t, T)$ is the discount factor at the value date. The above formulae are in a world that is forward risk-neutral with respect to a specific currency C_p . If an underlying asset i is measured in another currency C_U , the governing price dynamics of this underlying asset in the risk-neutral world of C_p should be written as

$$dS_i(t) = (r_U - q_i - \rho\sigma_U\sigma_s)S_i(t)dt + \sigma_s S_i(t)dW_t^i, \quad i = 1, \dots, n$$

where r_U is the short rate of C_U , q_i is the dividend yield of the asset, ρ is the correlation coefficient between the asset price and the cross-currency exchange rate, σ_s is the volatility of the asset price, σ_U is the volatility of the exchange price, and W_t^i is the Wiener process. Asset prices in the basket are correlated with $[dW_t^i, dW_t^j] = \rho_{i,j}dt$ where $\rho_{i,j}$'s are constant correlation coefficients between the logarithmic asset prices. All these parameters are assumed deterministic (see <https://finpricing.com/lib/FxAccumulator.html>).

To compute this expectation we use a Monte Carlo approach. First we will describe the **multi-asset path generation**. Let $W_i(t)$ be a 1-dimensional Brownian motion under currency C_i risk-neutral world, for all $i = 2, \dots, n$. Let $(WC_1(t), \dots, WC_n(t))$ be a n -dimensional Brownian motion under currency C risk-neutral world having (instantaneous) correlation matrix $(\rho_{ij})_{i,j=1,\dots,n}$, that is:

$$[dW_i^C(t), dW_j^C(t)] = \rho_{ij}dt \quad \forall i, j.$$

We have, for each $i = 2, \dots, n$:

$$dS_i(t) = (r_{C_i}(t) - q_i)S_i(t)dt + \sigma_{S_i}S_i(t)dW_i(t).$$

Repeating this procedure for all $k = 1, \dots, p$ (new standard Gaussian random vector for each date, but the same seed), we produce the averages $A_i, i = 1, \dots, n$ necessary in the payoff.

We compute the payoff M times and then take the average. If M is large enough, then these average approaches the expectation we are looking for (an application of the Strong Law of Large Numbers). Alongside we also compute the statistical standard error. Note that if the valuation date is later than an averaging date, then on that date the realized price must be used in the computation of the payoffs. The sensitivities are computed using finite differencing with path recycling. For example, delta is computed using:

$$\frac{\partial f}{\partial S} \sim \frac{f(S + \Delta S) - f(S - \Delta S)}{2 \cdot \Delta S},$$