Parametric Value At Risk

Value at Risk is the regulatory measurement for assessing market risk. It reports the maximum likely loss on a portfolio for a given probability defined as x% confidence level over N days.

Value at risk is vital in market risk management and control. Also regulatory and economic capital computation is based on Value at risk results. Although Value at risk measure is objective and intuitive, it doesn't capture tail risk.

There are three commonly used methodologies to calculate Value at risk – parametric, historical simulation and Monte Carlo simulation. This presentation focuses on parametric Value at risk.

Parametric Value at risk Methodology

- Assuming an asset return/valueChange follows normal distribution, the quantile of 99% confidence level is 2.33σ where σ is standard derivation

Parametric Value at risk Implementation

- For each asset/instrument (risk factor), calibrate volatility σ_i based on daily return
- For each pair, calibrate correlation ρ_{ij}
- Calculate the value at riskiance of a portfolio value change

$$V_p^2 = \lfloor \Delta(P_1)\sigma_1 \quad \dots \quad \Delta(P_n)\sigma_n \rfloor \begin{bmatrix} \rho_{11} & \cdots & \rho_{1n} \\ \vdots & & \vdots \\ \rho_{n1} & \cdots & \rho_{nn} \end{bmatrix} \begin{bmatrix} \Delta(P_1)\sigma_1 \\ \vdots \\ \Delta(P_n)\sigma_n \end{bmatrix}$$

- The portfolio Value at risk is $2.33\sqrt{V_p^2}$

Δ(P) = Delta * Δ(X) + 0.5 * Gamma * Δ(X)^2 + Vega*Δ(V)+Theta * Δ(T)

Value at risk Scaling

- Normally firms compute 1-day 99% Value at risk
- Regulators require 10-day 99% Value at risk
- Under IID assumption, 10-day Value at risk = $\sqrt{10} * VaR_{1-day}$

Reference:

https://finpricing.com/lib/FiBondCoupon.html